

**The nest site lottery:
how selectively neutral density dependent growth
suppression
induces frequency dependent selection**

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Abstract

Modern developments in population dynamics emphasize the role of the turnover of individuals. In the new approaches stable population size is a dynamic equilibrium between different mortality and fecundity factors instead of an arbitrary fixed carrying capacity. The latest replicator dynamics models assume that regulation of the population size acts through feedback driven by density dependent juvenile mortality. Here, we consider a simplified model to extract the properties of this approach. We show that at the stable population size, the structure of the frequency dependent evolutionary game emerges. Turnover of individuals induces a lottery mechanism where for each nest site released by a dead adult individual a single newborn is drawn from the pool of newborn candidates. This frequency dependent selection leads toward the strategy maximizing the number of newborns per adult death. However, multiple strategies can maximize this value. Among them, the strategy with the greatest mortality (which implies greater instantaneous growth rate) is selected. This result is important for the discussion about universal fitness measures and which parameters are maximized by natural selection. This is related to the fitness measures R_0 and r , because the number of newborns per single dead individual equals lifetime production of newborn R_0 in models without ageing. Our model suggests the existence of another fitness measure which is the combination of R_0 and r . According to the nest site lottery mechanism, at stable population size, selection favours strategies with the greatest r from those with the greatest R_0 .

Keywords: density dependence, logistic equation, frequency dependent selection, life history, evolutionary game, fitness measures

1. INTRODUCTION

In the current development of th evolutionary ecology (Post and Palkovacs 2009, Pelletier et al. 2009, Morris 2011, Schoener 2011) a problem of the eco-evolutionary feedback is of the special interest. One of the major theoretical problems in the modelling of population dynamics, and in general of evolutionary biology and ecology, is the limit of population growth and its selection consequences. This topic is very important in many disciplines such as evolutionary game theory and life history theory. The earliest attempt to solve this problem for populations with overlapping generations is the continuous logistic equation introduced by Verhulst in the 19th century (Verhulst, 1838), which can be found in every textbook on ecology and mathematical biology. It inspired the idea of r and K selection (McArthur and Wilson 1967), that selection favours different strategies at low densities and near the stable population size, and is still applied in modelling (Cressman et al. 2004, Cressman and Krivan 2006, Cressman and Krivan 2010). This concept states that there is some arbitrary maximal population size at which growth is suppressed and the population remains stable. However, this approach produces some unusual predictions which provoked a wide discussion (Kozłowski 1980, Łomnicki 1988, Kuno 1991, Ginzburg 1992, Gabriel 2005, Hui 2006, Argasinski and Kozłowski 2008) presented in the next section. The problem of the limits to growth is important not only for ecological population growth models but also for the modelling of natural selection. There is huge discussion on what is maximized by natural selection and what happens when a population reaches the limit to growth (Metz et al. 1992, Kozłowski 1993, Mylius and Dieckmann 1995, Brommer 2000, Dieckmann and Metz 2006, Metz et al. 2008, Roff 2008). However in these attempts density dependence

is represented by some abstract unspecified factors. Thus the proposed solutions are very general and abstract. A concrete mechanistic interpretation should be helpful in the interpretation of the mathematical notions. In the modification of the logistic equation (Kozłowski 1980, Hui 2006) which was applied in game-theoretic modelling (Argasinski and Kozłowski 2008, Argasinski and Broom 2012) there is an example of a mechanism responsible for strategically neutral density dependence called in this paper a "nest site lottery". The underlying assumption is that there is a limited number of nest sites at the environment and that every newborn must find a nest site to survive. Thus all newborns produced in some time moment form a pool of candidates to be drawn from to replace the dead individuals in their nest sites. The difference is that in this case there is arbitrary maximal population size described by a carrying capacity indicating the number of available nest sites (mechanistically interpreted as a nests or holes, where individual can settle, as in Hui 2006). However, the stable population size is not the carrying capacity, as in the classical logistic equation, but the dynamic equilibrium between different factors of mortality and fecundity (Kozłowski 1980, Ginzburg 1992, Hui 2006), which can be affected by the dynamics of the population state (Argasinski and Broom 2012). The advantage of this approach is that it considers a realistic turnover of individuals (Argasinski and Kozłowski 2008). In this paper we will more rigorously analyze the properties of the nest site lottery mechanism in a simpler model than in the previous papers (Argasinski and Kozłowski 2008, Argasinski and Broom 2012).

Below we give the mathematical details of this approach (sections 2 and 3). Section 4 starts the development of the selection model and at section 5 the important

notion of the turnover coefficient will be introduced. Section 6 contains presentation of selection dynamics. Section 7 contains the main result which are equations (13,12) and their analysis describing the nest site lottery mechanism (intuitively depicted at Fig. 1). The mechanistic reasoning from section 7 is completed by Theorem 1 describing quantitative characterization of the restpoints of the system (13,12). In section 8 the relationships between the nest site lottery mechanism and the invasion fitness concept are presented. The importance of the obtained results and general ideas inspired by them are presented in the discussion (the last subsection contains discussion on the existence of the new fitness measure suggested by obtained results).

2. DENSITY DEPENDENCE

The cornerstone of mathematical ecology is the Malthusian equation describing exponential population growth,

$$(1) \quad \frac{dn}{dt} = nr = nb - nd = n(b - d),$$

where b is the number of newborns and d describes the fraction of dead individuals per unit time. Thus the Malthusian coefficient r can be interpreted as the balance between mortality d and fertility b . The above model is not realistic, because it allows for infinite population growth. The classical solution of this problem is the use of the logistic equation, which is equation (2),

$$(2) \quad \frac{dn}{dt} = nr \left(1 - \frac{n}{K}\right).$$

However, this relies on a problematic assumption which has very serious consequences. Equation (2) for example produces artifacts in population growth models

(Kuno 1991, Gabriel 2005) and selection models related to replicator dynamics (Argasinski and Kozłowski 2008). For example, it suppresses the selection dynamics in the replicator dynamics by setting the right hand sides of equations to 0 (Argasinski and Kozłowski 2008), the trajectory escapes to infinity for $r < 0$ (i.e. $b < d$) and initial size greater than K (known as Levins' paradox, (Gabriel 2005)) or the trajectory decreases with increasing rate for $r < 0$ and initial size slightly smaller than K (Kuno 1991). This is caused by the fact that the whole term r is multiplied by the suppression coefficient, which implies that with population growth, both mortality and fertility decrease, and mortality decreasing with increasing population size and reaching zero at equilibrium is biologically counterintuitive. Mortality should not decrease with population growth and individuals cannot be immortal at equilibrium. The above problems suggest that models should rely on clear and mechanistic assumptions (Geritz and Kisdi 2012). Thus, density dependent suppression should act only on the number of juveniles recruited to the population (Kozłowski 1980, Ginzburg 1992) and the initial population size should be smaller than the carrying capacity (Hui 2006, Argasinski and Kozłowski 2008, Argasinski and Broom 2012) leading to

$$(3) \quad \dot{n} = n \left(b \left(1 - \frac{n}{K} \right) - d \right).$$

where suppression $(1 - n/K)$ describes newborns survival. This implies important distinction between newborn candidates introduced to the environment (described by b) and recruited newborns, survivors of density dependent stage (described by $b \left(1 - \frac{n}{K} \right)$). This problem was emphasized by Kozłowski (1980) for the first time, but surprisingly this paper did not get as wide an appreciation as it deserved. Then it was reinvented by Ginzburg (1992), but (3) was rejected there as it

“disagrees with our intuition about unchanging equilibrium ”. Hui (2006) argued, against Ginsburg’s claim, that (3) is the proper approach and should substitute for (2). The discussion started by Ginzburg also did not receive wide attention. Without knowledge of this discussion, (3) was applied to avoid the suppression of selection after reaching the equilibrium size caused by (2) (Argasinski and Kozłowski 2008) and is a cornerstone of the ecologically realistic approach to dynamic evolutionary games (Argasinski and Broom 2012). Then (3) was mentioned as an example of the proper mechanistic approach at (Geritz and Kisdi 2012), but not as the general alternative to (2). However, we believe that (3) deserves much stronger attention from a general audience. Although (3) has been applied in complex selection models (Argasinski and Kozłowski 2008, Zhang and Hui 2011, Argasinski and Broom 2012), the selection consequences of this approach were not rigorously analyzed, since previous papers (Kuno 1991, Ginzburg 1992, Gabriel 2005, Hui 2006) were focused on population density dynamics and ecological aspects. The selection mechanism induced by (3) is very interesting and will be analyzed in the further sections.

3. THE POPULATION IN EQUILIBRIUM

We can calculate equilibrium size, by setting the right hand side of Equation (3) to be equal to 0, which gives either $n = 0$ or

$$(4) \quad \tilde{n} = \left(1 - \frac{d}{b}\right) K.$$

Note that for positive \tilde{n} , the condition $b > d$ should be satisfied. After substitution of \tilde{n} into the logistic coefficient $(1 - n/K)$, we obtain the equilibrium newborn

survival d/b . This is reasonable; due to the turnover of individuals we have nb newborns and nd dead individuals per unit time. Thus nd/nb describes the number of newborns competing for single nest site vacated by a dead individual. Only one newborn can settle in a single place, thus each newborn can survive with probability d/b . This newborn survival should be valid for any density dependent mortality acting on juveniles, not only for logistic suppression, because only in this case does fertility equal overall mortality.

4. THE CASE OF MULTIPLE INDIVIDUAL STRATEGIES

Assume that there are different individual phenotypes $i = 1, \dots, H$ each characterized by reproduction b_i and mortality d_i . Thus every strategy is described by a two dimensional vector $v_i = [b_i, d_i] \in ([0, \infty) \times [0, 1])$.

Denoting $n = \sum_i n_i$ and $q_i = \frac{n_i}{n}$, we can describe the following dynamics:

$$(5) \quad \frac{dn_i}{dt} = n_i \left(b_i \left(1 - \frac{n}{K} \right) - d_i \right).$$

Thus

$$\frac{dn}{dt} = \sum_i \dot{n}_i = \sum_i n_i \left(b_i \left(1 - \frac{n}{K} \right) - d_i \right) = n \left(\left(1 - \frac{n}{K} \right) \sum_i q_i b_i - \sum_i q_i d_i \right),$$

giving

$$(6) \quad \frac{dn}{dt} = n \left(\left(1 - \frac{n}{K} \right) \bar{b} - \bar{d} \right),$$

where $\bar{b}(q) = \sum_i q_i b_i$ and $\bar{d}(q) = \sum_i q_i d_i$. We can easily calculate that in this case, instead of reaching the stable equilibrium, population size converges to the stationary density manifold (Cressman et al 2001, Cressman and Garay 2003a and

b)

$$(7) \quad \tilde{n} = \left(1 - \frac{\bar{d}}{b}\right) K.$$

which form is conditional on the strategy frequencies. Thus we introduced the diversity among individual strategies to our model. Newly produced offspring of the carriers of the different strategies forms a pool of candidates from which a randomly drawn individuals will be recruited to settle in the available nest sites. This is the core of the "nest site lottery" mechanism which will be analyzed in the next sections.

5. THE TURNOVER COEFFICIENT L

Here we will introduce an important characterization of the population dynamics. We shall define the function $L(v) = b/d$ for a single strategy v . L describes the number of newborns per single dead individual, which we shall refer to as the turnover coefficient (for the relationship of the turnover coefficient with lifetime reproduction, see Discussion). Surprisingly, a similar coefficient describing the energy allocation into reproduction divided by mortality can be found in life history papers (Taylor and Williams 1984, Kozłowski 1992, Kozłowski 1996, Werner and Anholt 1993, Perrin and Sibly 1993, for overview Kozłowski 2006). Analogously, for a mixture of strategies where $\bar{v}(q) = \sum_i q_i v_i = [\bar{b}, \bar{d}]$ is the average strategy contained in the convex hull of the strategies v_i (see figure 1), we define $L(\bar{v}(q)) = \bar{b}/\bar{d}$. Thus

$$(8) \quad L(\bar{v}(q)) = \frac{\bar{b}}{\bar{d}} = \frac{\sum q_i b_i}{\sum q_i d_i} = \frac{\sum q_i d_i L_i}{\sum q_i d_i} = \sum \frac{q_i d_i}{\sum q_j d_j} L_i = \sum y_i L_i$$

which is a weighted average of the L_i s and $y_i = q_i d_i / \sum q_j d_j$ describes the fraction of i strategists among individuals dying during a small time interval Δt (according to Appendix A, A.2). This is thus the average L_i among dead adult individuals. The L -function can be useful in describing the multiplicative newborn survival (recruitment probability) because after substitution of the stable population size \tilde{n} into the logistic suppression coefficient we obtain:

$$(9) \quad \left(1 - \frac{\tilde{n}}{K}\right) = \bar{d}/\bar{b},$$

which can be denoted as $1/L(\bar{v}(q))$.

If there is any variation in the L_i s, then we have that \bar{d}/\bar{b} lies strictly between the smallest and largest values of L , $L_{\min} < \bar{b}/\bar{d} < L_{\max}$. $L(v)$ describes the number of newborn candidates produced per single dead individual for the strategy $v = [b, d]$, during Δt . When the strategic argument is the averaged vector describing a population with a mixture of strategies, then the value of L is the average number of newborn candidates produced per single individual in this population. When the population is in size equilibrium (at the stationary density manifold), then the newborn survival component can be described by the value of L of the average population strategy, thus it becomes frequency dependent.

6. SELECTION DYNAMICS

Now we can describe the selection process realized by "nest site lottery" mechanism. Thus we should describe population in terms of the strategy frequencies q_i and the population size n . In our model, in the general case, newborn survival $\left(1 - \frac{n}{K}\right)$ is the phenomenological function, linear with respect to the fraction of free nest sites. However, at the stable size manifold it has the form (9). We can

assume that the strategies are enough close to each other that a separation of timescales between fast n dynamics and q dynamics occurs. Then we can assume that the selection occurs on the stationary size manifold. Thus using (9), we can write the selection dynamics from (5) as

$$(10) \quad \frac{dn_i}{dt} = n_i \left(b_i \frac{\bar{d}(q)}{b(q)} - d_i \right).$$

Because the average growth rate on the stable size manifold is zero then the equations (10) can be substituted by the replicator dynamics (see Appendix A)

$$(11) \quad \frac{dq_i}{dt} = q_i \left(b_i \frac{\bar{d}(q)}{b(q)} - d_i \right) = q_i d_i \left(\frac{L(v_i)}{L(\bar{v}(q))} - 1 \right).$$

Therefore the growth rate of the i -th strategy becomes a function of the strategy frequencies q (frequency dependent):

$$M(v_i, q) = b_i \frac{\bar{d}(q)}{b(q)} - d_i = d_i \left(\frac{L(v_i)}{L(\bar{v}(q))} - 1 \right),$$

and by equation (6) the population size manifold is described by

$$\tilde{n} = \left(1 - \frac{\bar{d}(q)}{b(q)} \right) K.$$

Note that the growth rate function M describes a mixture of all mortality and fecundity components, not only the density independent mortality d and fecundity b as in the Malthusian parameter r . The growth rate is positive when $L(v_i) > L(\bar{v}(q))$, which implies that

$$\frac{b_i}{d_i} > \frac{\sum_i q_i b_i}{\sum_i q_i d_i}.$$

Thus there is a threshold between regions of growth (strategies with reproductive surplus) and reduction (strategies with death rate exceeding birth rate) which has the linear form $b_i = L(\sum_i q_i v_i) d_i$ (see Figure 1). The threshold describes the set of

strategies for which the growth rate $M(v_i, q_i)$ equals 0. Frequencies q_i of strategies v_i with a greater value of L than the average strategy $\sum_i q_i v_i$ will increase under the replicator dynamics. In effect the averaged strategy shifts towards those strategies because it is a linear combination of the strategies present in the population. This implies an increase of L of the average strategy (see Figure 1). However, among growing strategies, the greatest growth rate is by the strategy with the greatest Malthusian parameter $M(v_i, q_i)$. Frequency dependence induces an increase of the slope of the threshold which eventually leads to the selection of the strategy with the greatest L , which confirms the result of Mylius and Diekmann (1995). Note that their second result, that density dependent adult mortality leads simply to r maximization as in unlimited growth models, directly comes from the independence of replicator dynamics from background fitness.

7. MULTIPLE STRATEGIES WITH $L_i = L_{\max}$

We have seen that evolution leads to the fixation of the strategy with the largest value of L , L_{\max} . What if there is more than one such strategy? The following question arises: is there selection between strategies with the same L ? We can show this by applying a multipopulation game-theoretic approach (Appendix A and Argasinski 2006) and divide strategies present in the population among subpopulations with the same L , but different ds (Appendix B). In effect we obtain the following equations

$$(12) \quad \frac{dq_i^j}{dt} = q_i^j \left(\frac{L(v^j)}{L(\bar{v}(q))} - 1 \right) \left(d_i^j - \sum_w q_w^j d_w^j \right),$$

$$(13) \quad \frac{dg_j}{dt} = g_j \left(\frac{L(v^j)}{L(\bar{v}(q))} - 1 \right) \sum_w q_w^j d_w^j,$$

describing the selection of strategies within the j -th L -class (equation (12)) and related frequencies between L -classes (13) described by g_j . Thus selection between L -classes is driven by the first bracketed term from Equation (12) and affects both intra- and inter-group dynamics. However, there is selection inside each L -class toward greater d . When suboptimal L -classes are outcompeted, the intrinsic selection driven by the bracket $\left(d_i^j - \sum q_w^j d_w^j\right)$ is also suppressed. The form of Equation (13) shows that among growing L -classes, those with smaller L can grow faster than those with larger, due to a greater $\sum q_w^j d_w^j$, until they fall under the L -selection threshold.

We are interested in the question which strategy (strategies) will dominate the population in the long term. In particular, \dot{q}_i is always positive if strategy i has $L_i = L_{\max}$, whenever there is variation in the L values in the population. Thus the proportions of such strategies increase; but also, following Equation (12), the strategies out of these with the largest values of d_i increase the fastest. Thus if there is either repeated small mutations involving strategies with $L_i < L_{\max}$ or a constant low level of mutation involving a mix of strategies making $L(\bar{v}(q)) < L_{\max}$, the population will evolve to the strategy out of those with $L_i = L_{\max}$ such that d_i takes the largest value. Thus repeated mutations or invasions of suboptimal strategies induce selection towards maximal d among L_{\max} strategists. It is easy to show that in the absence of density dependent suppression, this strategy has the greatest r but only among L_{\max} strategies, since $b_i = d_i L_{\max}$. The strategies from other L -classes can have even greater r , but they will be outcompeted by mechanism described by bracketed term in equation (11). In the case when population consists only of the L_{\max} individuals, the same outcome can be caused by repeated ecological

catastrophes leading to the decrease of the population size. Then the strategy with the greatest d_i will have the greatest growth rate during the growth phase of the population.

However, let us focus on the evolution of the system under the replicator dynamics in a single particular “turn”, during which no mutation occurs. Suppose that there are precisely I strategies in the L_{\max} -class. Assume that initial state of the L_{\max} -class is described by vector $q = (q_1^{\max}(0), \dots, q_I^{\max}(0))$ and initial relative size g_{\max} .

Theorem 1

The replicator dynamics converges to the vector $\mathbf{q} = (q_1^{\max}(0)g_{\max}(0)\lambda^{d_1}, \dots, q_I^{\max}(0)g_{\max}(0)\lambda^{d_I})$, where λ is a constant that satisfies the equation $g_{\max}(0) \sum_{i=1}^I q_i(0)\lambda^{d_i} = 1$.

For the proof see Appendix C.

The above theorem shows that the restpoint describing frequencies among L_{\max} strategists is fully determined by the initial state of the L_{\max} -class and its relative size. Despite frequency dependence, independently of the initial frequencies of the other strategies (note that in our model there are no direct interactions between individuals). Thus calculation of \mathbf{q} reduces to finding of the appropriate value of λ . Note that the rest point q can be interpreted as the state of the whole population in general coordinates and the final state of the L_{\max} -class (then $g_{\max} = 1$). If initial $g_{\max}(0) = 1$ then obviously $\lambda = 1$, thus parameter λ can be described as the inflation coefficient because it inflates the frequencies to sum them to one and compensate the impact of $g_{\max}(0)$.

8. THE MONOMORPHIC RESIDENT-MUTANT CASE

We can simplify the above model by assuming a monomorphic population invaded by a rare mutant; thus this resembles the classical ESS approach (Maynard Smith 1982, in the context of life history evolution Charlesworth & Leon 1976, Mylius and Dieckmann 1995). In the limit as the change of the strategy trait tends to zero, we approach the method known as invasion analysis which is the cornerstone of adaptive dynamics (Dieckmann and Law 1996, Metz et al. 1996, Geritz et al. 1998, Dercole and Rinaldi 2008). Using Equation (11), the resident Malthusian parameter is zero and the rare mutant Malthusian parameter is $(b_{mut}/b_{res})d_{res} - d_{mut}$ which must be positive to invade the population. Thus and the equilibrium population size increases.

To be an ESS itself, the “mutant” population should be stable against the previous resident, and thus:

$$\frac{b_{res}}{b_{mut}}d_{mut} - d_{res} < 0 \Rightarrow \frac{b_{res}}{d_{res}} < \frac{b_{mut}}{d_{mut}},$$

which is the same condition.

Therefore we have obtained for the “nest site lottery” mechanism, the classical result that under limited growth only lifetime reproduction is maximized and there is no selection pressure on the lifespan. However this occurs only in a monomorphic resident-mutant model.

9. DISCUSSION

9.1. How the nest site lottery works? We started from the basic population growth equation which is the cornerstone of the framework underlying evolutionary game theory and replicator dynamics (Maynard Smith 1982, Cressman 1992, Hofbauer and Sigmund 1988 and 1998) and its more ecologically realistic extensions (Cressman and Garay 2004, Argasinski 2006, Argasinski and Kozłowski 2008, Argasinski and Broom 2012). We presented an analysis of the dynamics of the mechanism inducing frequency dependent selection toward the strategy maximizing the turnover coefficient $L(v_i)$.

This phenomenon can be explained mechanistically. All newborns introduced to the population in the same time moment form a pool of candidates. Each newborn is independently drawn to survive (find a nest place), thus the strategy maximizing the number of newborns (trials) maximizes the fraction in the pool of candidates and in effect the amount of survivors. However, every dead adult can be substituted by any other strategy, thus each death is an additional free place in the lottery. Thus it is profitable for the strategy carried by some subpopulation to maximize the number of trials (newborns) per single offered place (dead adult). In addition, we have shown that among strategies with the largest value of the turnover coefficient L_{\max} there is a selection pressure toward the strategy with the greatest d . This is intuitive from equation (11), because for strategies with the maximal number of newborn candidates produced per each dead adult (i.e. maximizing the bracketed term in equation (11)), the growth rate will increase with the number of dead adults (described by the fraction $q_i d_i$ in the equations (11)) since each of them will be exchanged for L_{\max} newborns in the pool of candidates. Note that

this provides a gene centered mechanistic explanation of the phenomenon which can be naively interpreted in terms of group selection and altruistic "sacrifice" of adults, to release the nest sites for juveniles. We show that it is the tradeoff between maximization of the number of candidates in the nest site lottery and survival of parental individual.

9.2. Importance of the nest site lottery mechanism. The model presented in this paper is as simple as possible, to emphasize the mechanistic aspects of the analyzed phenomenon. For example, there are no direct interactions between individuals as in game-theoretic models. However, it was shown that the impact of density dependent factors (thus also the mechanism described in this paper) can significantly affect and alter the outcomes of game-theoretic models (Argasinski and Kozłowski 2008, Argasinski and Broom 2012). This is caused by the feedback driven by the fact that on stable size manifold every newborn should find a new nest site vacated by a dead adult. Our model is the simple case example of the one-dimensional monotone density dependence acting on effective birth rate (more on this and other general cases can be found at Metz et al 2008). Our model has extremely simplified age structure consisting only of juveniles and adults (more on the limitations of pure age-dependent models can be found at Metz and Diekmann 1986). However, it is possible that other environmental feedback loops of the same type than in our model may induce similar selection mechanisms. Maybe phenomena similar to those revealed by our simple model, can be found in other, more general or different specific models related to general population dynamics, life history evolution, adaptive dynamics or population genetics. This can be subject of the future research.

The mechanism shown in this paper supports the intuition underlying the r and K selection theory (McArthur and Wilson 1967), that natural selection favours different strategies in growing populations than in populations with suppressed growth. The theoretical and methodological aspects of this approach were criticized (Barbault 1987, Getz 1993, Stearns 1977), however as an intuition it still seems to be relevant (for modern approaches see for example Metz et al. 2008). The alternative for r and K approach is the life history theory (Roff 1992, Stearns 1992), where the problem of different selection mechanisms in limited and unlimited populations also exists.

9.3. What is maximized by natural selection, and when? The exact meaning of “fitness” is a subject of endless discussion (Metz et al. 1992, Kozłowski 1993, Mylius and Diekmann 1995, Brommer 2000, Dieckmann and Metz 2006, Metz et al. 2008, Roff 2008). Basically “fitness” can be defined as the instantaneous growth rate or invasion exponent (Metz 2008c). However, if eco-evolutionary feedback is of particular simple kind, the optimization approach can be applied (Metz et al. 2008, Gyllenberg et al. 2011) where some “fitness measures” or “proxies” are maximized. There is a widely known fact in life history theory that in a population with unlimited growth, the Malthusian parameter r is a proper fitness measure, while on stable size manifold, lifetime production of newborns (before juvenile mortality selection) R_0 is the correct measure. In (Mylius and Diekmann 1995) there is a statement that the invasion fitness method (Metz et al. 1992) suggests that R_0 and r are necessarily both maximal at the ESS (although this statement is unclear since some strategies can maximize R_0 and others can maximize r). Our results support

this claim, and show that an analogous mechanism can act in population dynamic and game-theoretic models.

Here an important claim is that of Brommer and Kokko (2002), who say that R_0 is a rate independent reproductive measure which does not depend on the timing of reproductive events. This is because R_0 is described on the lifespan timescale, not the population dynamic timescale like r . Despite simplified form our model can be useful illustrative example for this problem. How can we use a lifespan perspective in our approach? At first, assume that the population growth is unlimited (every newborn candidate can find a nest site). Then demographic parameters b and d are constant. When we consider only the death component of the Malthusian equation then we obtain the equation $\dot{n} = -nd$; thus this is an exponential decay with decay constant d , and so the average lifetime of the individual is $\tau = 1/d$. Then $r = (L - 1)/\tau$ and $L(v)$ is the lifetime reproduction R_0 (or the average R_0 among individuals dying during Δt in a population described by v). Therefore, the formula $r = (L - 1)/\tau$ shows how the growth rate is affected by lifetime reproduction in the case of a non-age structured population with unlimited growth. It shows that an individual should basically replace itself, but for strategy growth rate to be positive requires a reproductive surplus during a lifetime. Now introduce the limitation of the nest sites causing density dependent selection. Our result shows that under density dependence, the growth rates of the strategies are affected by the frequency dependent surplus reducing mechanism, described by newborn survival $1/L(\bar{v})$ which is the function of the other strategies present in the population. In

effect $r(v) = (L(v)/L(\bar{v}) - 1) / \tau$ and $R_0 = L(v)/L(\bar{v})$. Thus the strategy maximizing the lifetime production of newborn candidates $L(v)$ will maximize R_0 and the bracketed term of $r(v)$.

9.4. Conclusion. Our simple model suggests an insight into the mechanistic nature of this phenomenon and has serious interpretational consequences. It clearly shows that this problem should not be formulated as the alternative: evolution maximizes r OR R_0 . In our simple model, when the population reaches a stable size manifold, then a mechanism that modifies the r 's of competing strategies, which are no longer constants, emerges to select the strategy with maximal R_0 , or with maximal r , among multiple strategies with maximal R_0 . Thus, our model suggests the existence of another fitness measure which is the combination of R_0 and r , if our reasoning holds in age structured and other more complex models. However, it will be not a function which should be maximized, but a two staged procedure. First stage should identify the strategies maximizing the turnover coefficient, while second stage should find strategies with the greatest r from strategies chosen in the first stage. We note that our analysis is a simplification, and whereas $R_0 = L(v_i)$ in the models without age structure as presented in this paper, this is not necessarily satisfied in age structured models. This should be the subject of future research.

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Appendix A Multipopulation replicator dynamics.

Assume that we have u individual strategies. Standard replicator dynamics can be derived by rescaling the Malthusian growth equations $\frac{dn_i}{dt} = n_i M_i$ to the related frequencies $q_i = n_i / \sum_j n_j$ which leads to equations $\frac{dq_i}{dt} = q_i [M_i - \bar{M}]$ (where $\bar{M} = \sum_j q_j M_j$). Those equations describe the evolution of strategy frequencies in the unstructured population. However, we might be interested in the modelling of the structured population divided into subpopulations such as different sexes, species etc. Assume that we want to decompose an entire population into z subgroups. Define

$$(A.1) \quad k^j = [k_1^j, \dots, k_{u_j}^j]$$

as a vector of indices of strategies exhibited by individuals from the i -th subgroup ($k_i^j \in \{1, \dots, u\}$, and u_j is the number of strategies in the i -th subgroup). For example the notation $k^2 = [1, 3, 5]$ means that, in the second subgroup there are (only) individuals with strategies 1, 3 and 5. Every strategy should belong to a single unique subgroup. Then according to Argasinski (2006), by the following change of coordinates

$$(A.2) \quad q^j = [q_1^j, \dots, q_{u_j}^j] = \left[\frac{q_{k_1^j}}{\sum_{i=1}^{u_j} q_{k_i^j}}, \dots, \frac{q_{k_{u_j}^j}}{\sum_{i=1}^{u_j} q_{k_i^j}} \right] \quad j = 1, \dots, z$$

we obtain a distribution of relative frequencies of strategies in the i -th subpopulation. The distribution of proportions between subpopulations has the form

$$(A.3) \quad g = [g_1, \dots, g_z] = \left[\sum_{i=1}^{u_1} q_{k_i^1}, \dots, \sum_{i=1}^{u_z} q_{k_i^z} \right],$$

where g_j is the proportion of the j -th subpopulation. Every decomposition into subpopulations can be reduced again to a single population model by the opposite change of coordinates $q(g, q^1, \dots, q^z)$ where

$$(A.4) \quad q_{k_i^j} = g_j q_i^j.$$

When we apply the above transformations to the replicator equations, we obtain a set of equations that describes the dynamics inside the subpopulations (intraspecific dynamics). When the set of strategies in each subpopulation is characterized by the vector of indices k^j , then the system of replicator equations will be:

$$(A.5) \quad \frac{dq_i^j}{dt} = q_i^j [M_i^j - \bar{M}^j] \quad i = 1, \dots, u_j - 1, \quad j = 1, \dots, z$$

$$(A.6) \quad \frac{dg_s}{dt} = g_s [\bar{M}^s - \bar{M}] \quad s = 1, \dots, z - 1$$

where $\bar{M}^s = \sum_{i=1}^{u_s} q_i^s M_i^s$ is the mean fitness in the s -th subpopulation and $\bar{M} = \sum_{s=1}^z g_s \bar{M}^s$. In practical applications of this method to the modelling of biological problems, the replicator equations can be defined on the decomposed population. This will simplify the formulation of the model, because when strategies are initially assigned to subpopulations, there is then no need to change their indices. The choice of subpopulations is arbitrary and depends on the biological assumptions underlying the analyzed problem. For example, the entire population may be divided into two competing subpopulations of hosts and parasites or prey and predators. On the other hand, it may be divided into two subpopulations of males and females, when interspecific dynamics will describe the evolution of the secondary sex ratio,

and intraspecific dynamics will describe changes of the frequencies of strategies inside the male and female subpopulations. The subpopulations can be divided into subsubpopulations, and the entire population may be transformed into a complex multilevel cluster structure. However, all these structures are equivalent to a single population replicator dynamics model.

Appendix B Derivation of equations (12) and (13) describing selection strategies inside L -classes and change of sizes of L -classes

Let us assume than we have m such classes. Then the initial system of the replicator equations can be transformed into two sets of differential equations. Firstly, the within L -class dynamics (according to A.5):

$$(B.1) \quad \frac{dq_i^j}{dt} = q_i^j \left(M(v_i^j) - \bar{M}^j \right),$$

where q_i^j is the proportion of the i -th strategy in the j -th L -class and $\bar{M}^j = \sum_w q_w^j M(v_w^j) = \sum_w q_w^j d_i (L(v_w^j)/L(\bar{v}(q)) - 1)$. Secondly, the between L -class dynamics (according to A.6):

$$(B.2) \quad \frac{dg^j}{dt} = g^j (\bar{M}^j - \bar{M}),$$

where g^j is the proportion of j -th L -class and $\bar{M} = 0$, since the population is on the stable size manifold. Since for all strategies (for all i) from the same L -class the term $(L(v_i^j)/L(\bar{v}(q)) - 1)$ in B.1 is the same within the class, the lower index i (representing the strategy number within that class) can be neglected. After substitution of the respective formulae into Equations (B.1) and (B.2), we obtain

the equations (12) and (13):

$$\frac{dq_i^j}{dt} = q_i^j \left(d_i \left(\frac{L(v_i^j)}{L(\bar{v}(q))} - 1 \right) - \sum_w q_w^j d_i \left(\frac{L(v_w^j)}{L(\bar{v}(q))} - 1 \right) \right) \quad (\text{B.3})$$

$$= q_i^j \left(\frac{L(v^j)}{L(\bar{v}(q))} - 1 \right) \left(d_i^j - \sum_w q_w^j d_w^j \right), \quad (\text{B.4})$$

$$\frac{dg_j}{dt} = g_j \left(\frac{L(v^j)}{L(\bar{v}(q))} - 1 \right) \sum_w q_w^j d_w^j. \quad (\text{B.5})$$

Appendix C: Proof of Theorem 1

From (11) we have that

$$\frac{dq_i}{dt} = d_i q_i \left(\frac{L(p_i)}{L(\bar{p}(q))} - 1 \right)$$

and so

$$(C.1) \quad \left(\frac{L(v_i)}{L(\bar{v}(q))} - 1 \right) = \frac{1}{d_i q_i} \frac{dq_i}{dt}.$$

Consider any pair of strategies $v_i = [b_i, d_i]$ and $v_j = [b_j, d_j]$ from the same L -class

(i.e. $L(v_i) = L(v_j)$). Using (C.1) we obtain

$$(C.2) \quad \begin{aligned} \frac{1}{d_i q_i} \frac{dq_i}{dt} &= \frac{1}{d_j q_j} \frac{dq_j}{dt} \Rightarrow \\ \int \frac{1}{d_j q_j} dq_i &= \int \frac{1}{d_j q_j} dq_j + C \Rightarrow \\ \frac{\ln q_i(t)}{d_i} &= \frac{\ln q_j(t)}{d_j} + C. \end{aligned}$$

Considering $t = 0$ in equation (C.2) we obtain

$$(C.3) \quad C = \frac{\ln q_i(0)}{d_i} - \frac{\ln q_j(0)}{d_j}.$$

Combining (C.3) with (C.2) we obtain

$$\frac{\ln q_i(t) - \ln q_i(0)}{d_i} = \frac{\ln q_j(t) - \ln q_j(0)}{d_j} \Rightarrow$$

$$(C.4) \quad \left(\frac{q_i(t)}{q_0(t)} \right)^{1/d_i} = \left(\frac{q_j(t)}{q_0(t)} \right)^{1/d_j}.$$

Equation (C.4) holds for any pair i, j so that

$$\begin{aligned} \left(\frac{q_i(t)}{q_0(t)} \right)^{1/d_i} &= \lambda(t) \Rightarrow \\ q_i(t) &= q_i(0) \lambda(t)^{d_i} \quad i = 1, \dots, I, \end{aligned}$$

for some $\lambda(t)$. Letting $\lambda = \lim_{t \rightarrow \infty} \lambda(t)$ gives

$$(C.5) \quad q_i = \lim_{t \rightarrow \infty} q_i(t) = q_i(0) \lambda^{d_i}.$$

We know that $\sum_i q_i = 1$, thus for at least one L -class respective $\lambda(t)$ should not converge to 0. System (12),(13) shows that it will be L_{\max} -class. However, above reasoning was described in the coordinates describing the strategy frequencies in the whole population (a metasimplex coordinates, Argasinski 2006). According to (A.4), $q_u(0)$ can be described in the coordinates of the system (12) and (13) and after change of the indexes $u = k_a^l$ where l is the index of the L -class and a is the index of the strategy within this L -class, we have $q_{k_a^l} = g_l q_a^l$. Thus the restpoint will contain only the L -maximizing strategies i.e state of the L_{\max} -class will be equivalent to the state of the whole population (i.e according to (A.4). $g_{\max} = 1$ and $q_{k_i^{\max}} = q_i^{\max}$), but frequencies $q_{k_i^{\max}}(0)$ will not sum to 1. However, from (A.4) we have $q_{k_a^{\max}}(0) = g_{\max}(0) q_a^{\max}(0)$. Then (C.5) for the L_{\max} -class can be presented as:

$$(14) \quad q_a^{\max} = q_a^{\max}(0) g_{\max}(0) \lambda^{d_a}.$$

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Figure legends:

Figure 1

The figure shows the plot of the L -surface over the strategy space. The individual strategies and L -classes are shown on the strategy space. The L -class of the populations average strategy is the border between regions of growth and decline. The growth of the frequencies of the strategies from the growth area induce counterclockwise movement of the L -class of the average strategy, leading to the selection of L -maximizing strategies. However, among L -maximizers, selection for greater mortality d can be induced by the introduction of sub-optimal mutants. In this case the strategy lying on the maximal L -class line, farthest from zero, wins.

